Stability of giant gravitons with NSNS B field

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Abstract

We study the stability of the giant gravitons in the string theory background with NSNS B field. We consider the perturbation of giant gravitons formed by a probe D$(8-p)$ brane in the background generated by D$(p-2)$-D(p) branes for $2 \le p \le 5$. We use the quadratic approximation to the brane action to find the equations of motion. The vibration modes for ρ , ϕ and r are coupled, while those of x_k's$(k = 1, ..., p-2)$ are decoupled. For $p = 5$, they are stable independent of the size of the brane. For $p \neq 5$, we calculated the range of the size of the brane where they are stable.

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I. INTRODUCTION

Stable extended brane configurations in some string theory background, called giant gravitons, attracted interests in connection with the stringy exclusion principle recently. Myers [1] found that certain D-branes coupled to RR potentials can expand into higher dimensional branes. McGreevy, Susskind and Toumbas [2] have shown that a massless particle with angular momentum on the S^n part of $\mathrm{AdS}_m \times \mathrm{S}^n$ spacetime blows up into a spherical brane of dimensionality $n-2$. Its radius increases with increasing angular momentum. The maximum radius of the blown-up brane is equal to the radius of the sphere that contains it since the angular momentum is bounded by the radius of S^n. This is a realization of the stringy exclusion principle [3] through the AdS/CFT correspondence [4]. Later it was shown that the same mechanism can be applied to spherical branes on the AdS part [5,6]. However, they can grow arbitrarily large since there is no upper bound on the angular momentum. To solve this puzzle, instanton solutions describing the tunneling between the giant gravitons on the AdS part and on the S part were introduced [5,7]. Giant graviton configurations preserving less than half of the supersymmetry were studied by Mikhailov [8]. A magnetic analogue of the Myers effect was investigated by Das, Trivedi and Vaidya [9]. They suggested that the blowing up of gravitons into branes are possible on some backgrounds other than $\mathrm{AdS}_m \times \mathrm{S}^n$ spacetime.

More recently it is known by Camino and Ramallo [10] that the giant graviton configurations are also possible in a string background with NS-NS B field. They considered the geometry formed by a stack of non-threshold bound state of the type $(\mathrm{D}(p-2),\, \mathrm{D}p)$ for $2 \le p \le 6$ [11], which is characterized by the non-zero Kalb-Ramond field B from the NS sector together with the corresponding RR fields. In this background they put a probe brane such that it could capture both the RR flux and the flux of B field. The probe brane and the branes of the background have two common directions. The probe brane is a $\mathrm{D}(8-p)$ brane wrapped on an S^{6-p} sphere transverse to the background and extended along the plane parallel to it. They showed that, for a particular choice of the worldvolume gauge field, one can find configurations of the probe brane which behave as massless particles and they can be interpreted as giant gravitons.

One important issue related to the giant gravitons is whether they are stable or not under the perturbation around their equilibrium configurations. Perturbation of the giant gravitons was studied first by Das, Jevicki and Mathur [12]. Using the quadratic approximation to the action, they computed the natural frequencies of the normal modes for giant gravitons in $\mathrm{AdS}_m \times \mathrm{S}^n$ spacetime for both cases when gravitons are extended in AdS subspace and they are extended on the sphere S^n. All modes have real positive ω^2 for any size of the branes so that they are stable. Perturbation analysis of giant gravitons whose background geometry is not of a conventional form of $\mathrm{AdS}_m \times \mathrm{S}^n$ was considered by the author [13]. The normal modes of giant gravitons in the dilatonic D-brane background were found and they turned out to be stable too.

In this paper, we will study the stability of giant gravitons in the string background with NS-NS B field described in Ref. [10]. We consider the perturbation of giant gravitons in the near-horizon geometry. In the previous analysis, the perturbation of the brane along the transverse direction was not considered. Here we consider the perturbation of this variable too. The organization of the paper is as follows. In Sec. II we review the giant gravitons with NSNS B field and set up some preliminaries for our calculation. In Sec. III we consider the perturbation up to second order and derive the equations of motion from which one determines the normal modes. From these equations we will discuss the stability of the giant gravitons. Finally in Sec. IV, we conclude and discuss our results.

II. BRANE ACTION

A. D$(p-2)$-Dp brane backgrounds

Consider the supergravity background generated by a stack of N non-threshold bound states of Dp and D$(p-2)$ branes for $2 \le p \le 6$. The metric and dilaton are given by [10,11]

$$ds^2 = f_p^{-1/2}[-(dx^0)^2 + \cdots + (dx^{p-2})^2 + h_p\{(dx^{p-1})^2 + (dx^p)^2\}] + f_p^{1/2}[dr^2 + r^2 d\Omega^2_{8-p}], \quad (1)$$

$$e^{\tilde{\phi}_D} = f_p^{\frac{3-p}{4}} h_p^{1/2}, \quad (2)$$

where $d\Omega^2_{8-p}$is the line element of S^{8-p}, r is the radial coordinate parametrizing the distance to the brane bound state and $\tilde{\phi}_D = \phi_D - \phi_D(r \to \infty)$. The metric of S^{8-p} can be written as

$$d\Omega^2_{8-p} = \frac{1}{1-\rho^2}d\rho^2 + (1-\rho^2)d\phi^2 + \rho^2 d\Omega^2_{6-p}, \quad (3)$$

where $d\Omega^2_{6-p}$ is the metric of a unit $6-p$ sphere. The range of the variable ρ and ϕ are $0 \le \rho \le 1$ and $0 \le \phi \le 2\pi$. The coordinate ρ plays the role of the size of the system on S^{6-p}. The Dp brane of the background extends on $x^0 \cdots x^p$, while the D$(p-2)$ brane lies along $x^0 \cdots x^{p-2}$. The functions f_p and h_p in Eqs. (1) and (2) are given by

$$f_p = 1 + \frac{R^{7-p}}{r^{7-p}},$$
$$h_p^{-1} = \sin^2\varphi f_p^{-1} + \cos^2\varphi, \quad (4)$$

where φ is the angle characterizing the degree of mixing of the Dp and D$(p-2)$ branes. The parameter R is given by

$$R^{7-p}\cos\varphi = N g_s 2^{5-p}\pi^{\frac{5-p}{2}}(\alpha')^{\frac{7-p}{2}}\Gamma(\frac{5-p}{2}), \quad (5)$$

where N is the number of branes of the stack, g_s is the string coupling constant ($g_s = e^{\phi_D(r\to\infty)}$) and α' is the Regge slope.

This solution also has a rank two NSNS B field on the $x^{p-1}x^p$ (noncommutative) plane

$$B = \tan\varphi f_p^{-1} h_p dx^{p-1} \wedge dx^p, \quad (6)$$

and is charged under RR field strengths, $F^{(p)}$ and $F^{(p+2)}$. The components along the directions parallel to the bound state are:

$$
\begin{aligned}
F^{(p)}_{x^0,x^1,\cdots,x^{p-2},r} &= \sin\varphi \partial_r f_p^{-1}, \\
F^{(p+2)}_{x^0,x^1,\cdots,x^{p},r} &= \cos\varphi h_p \partial_r f_p^{-1}.
\end{aligned} \tag{7}
$$

Note that $F^{(p)}$'s for $p \geq 5$ are the hodge duals of those with $p \leq 5$, i.e. $F^{(p)} =^* F^{(10-p)}$. For $\varphi = 0$ the (D$(p-2)$,Dp) solution reduces to the Dp brane geometry whereas for $\varphi = \pi/2$ it is a D$(p-2)$ brane smeared along the $x^{p-1}x^p$ directions.

The hodge duals of the RR field strengths can be easily computed from Eqs. (7) and (1)

$$
\begin{aligned}
{}^*F^{(p)}_{x^{p-1},x^p,\rho,\phi,\theta^1,\cdots,\theta^{6-p}} &= (-1)^{p+1}(7-p)\sin\varphi R^{7-p}\rho^{6-p}h_p f_p^{-1}\sqrt{\hat{g}^{(6-p)}}, \\
{}^*F^{(p+2)}_{\rho,\phi,\theta^1,\cdots,\theta^{6-p}} &= (-1)^{p+1}(7-p)\cos\varphi R^{7-p}\rho^{6-p}\sqrt{\hat{g}^{(6-p)}}.
\end{aligned} \tag{8}
$$

where $\theta^1, \cdots, \theta^{6-p}$ are the coordinates of S^{6-p} and $\hat{g}^{(6-p)}$ is the determinant of S^{6-p} metric. These forms satisfy

$$
d^*F^{(p)} = H \wedge^* F^{(p+2)}, \qquad d^*F^{(p+2)} = 0. \tag{9}
$$

And we can represent ${}^*F^{(p)}$ and ${}^*F^{(p+2)}$ in terms of RR potentials as follows:

$$
\begin{aligned}
{}^*F^{(p)} &= dC^{(9-p)} - H \wedge C^{(7-p)}, \\
{}^*F^{(p+2)} &= dC^{(7-p)} - H \wedge C^{(5-p)},
\end{aligned} \tag{10}
$$

where $H = dB$. Only for $p = 3$ the term $H \wedge C^{(5-p)}$ in the second equation gives a non-vanishing contribution. The components of the two potentials $C^{(7-p)}$ and $C^{(9-p)}$ are

$$
\begin{aligned}
C^{(7-p)}_{\phi,\theta^1,\cdots,\theta^{6-p}} &= (-1)^{p+1}\cos\varphi R^{7-p}\rho^{7-p}\sqrt{\hat{g}^{(6-p)}}, \\
C^{(9-p)}_{x^{p-1},x^p,\phi,\theta^1,\cdots,\theta^{6-p}} &= (-1)^{p+1}\sin\varphi R^{7-p}\rho^{7-p}h_p f_p^{-1}\sqrt{\hat{g}^{(6-p)}}.
\end{aligned} \tag{11}
$$

B. Brane motion

Now we embed a probe D$(8-p)$ brane in the near-horizon region of the (D$(p-2)$,Dp) geometry where r is small. In this region we can approximate the harmonic function f_p as

$$
f_p \simeq \frac{R^{7-p}}{r^{7-p}}. \tag{12}
$$

The probe D$(8-p)$ brane wraps the $(6-p)$ transverse sphere and extends along the $x^{p-1}x^p$ directions. The action for this case, ignoring the fermions, is given by the sum of a Dirac-Born-Infeld(DBI) and Wess-Zumino(WZ) terms

$$
S = S_{DBI} + S_{WZ}. \tag{13}
$$

The DBI action S_{DBI} is

$$
S_{DBI} = -T_{8-p}\int d^{9-p}\xi e^{-\tilde{\phi}_D}\sqrt{-\det(g+\mathcal{F})}, \tag{14}
$$

where g is the induced metric obtained by the pullback

$$g_{\alpha\beta} = \frac{\partial x^M}{\partial \xi^\alpha}\frac{\partial x^N}{\partial \xi^\beta} G_{MN}, \tag{15}$$

T_{8-p} is the tension of the D$(8-p)$ brane

$$T_{8-p} = (2\pi)^{p-8}(\alpha')^{\frac{p-9}{2}} g_s^{-1}, \tag{16}$$

and $\mathcal{F}$ is given by

$$\mathcal{F} = F - P[B] = dA - P[B]. \tag{17}$$

Here $P[\cdots]$ denotes the pullback of a bulk field to the worldvolume and F is a $U(1)$ worldvolume gauge field strength and A its potential. The Wess-Zumino action S_{WZ} is given by

$$S_{WZ} = T_{8-p}\int d^{9-p}\xi \Big\{ P[C^{(9-p)}] + \mathcal{F}\wedge P[C^{(7-p)}]\Big\}. \tag{18}$$

We take the worldvolume coordinates $\xi^\alpha (\alpha = 0, 1, \cdots, 8-p)$ in the static gauge as

$$\xi^\alpha = (t, x^{p-1}, x^p, \theta^1, \cdots, \theta^{6-p}). \tag{19}$$

The choice of coordinates Eq. (19) is convenient for the study of the background configurations in which we are interested. These configurations are the embeddings of the D$(8-p)$ brane where the dynamical variables are described by

$$r = r(t), \qquad \rho = \rho(t), \qquad \phi = \phi(t). \tag{20}$$

We evaluate the probe brane action under the ansatz of Eq. (20). For the Wess-Zumino term, only the components of $C^{(7-p)}$ and $C^{(9-p)}$ in Eq. (11) contribute. The pullback of the RR potential $C^{(7-p)}$ is coupled to the $x^{p-1}x^p$ component of $\mathcal{F}$. Assuming $\mathcal{F}_{x^{p-1},x^p}$ is independent of the angles $\theta^1\cdots\theta^{6-p}$, one can write S_{WZ} as

$$S_{WZ} = T_{8-p}\Omega_{6-p}R^{7-p}\cos\varphi\int dtdx^{p-1}dx^p\rho^{7-p}(-1)^{p+1}\dot{\phi}\Big(\mathcal{F}_{x^{p-1},x^p} + h_p f_p^{-1}\tan\varphi\Big), \tag{21}$$

where $\dot{\phi} = d\phi/dt$ and Ω_{6-p} is the volume of the unit $6-p$ sphere given by

$$\Omega_{6-p} = \frac{2\pi^{\frac{7-p}{2}}}{\Gamma(\frac{7-p}{2})}. \tag{22}$$

Note that the dynamical variable r, ρ, ϕ do not depend on the coordinates x^{p-1} and x^p, thus $P[B]$ has non-zero components along the $x^{p-1}x^p$ directions. Substituting the explicit form of B field in Eq. (6) into the definition of $\mathcal{F}$ (Eq. (17)), one gets

$$\mathcal{F}_{x^{p-1},x^p} + h_p f_p^{-1}\tan\varphi = F_{x^{p-1},x^p}. \tag{23}$$

In what follows we assume that the only non-zero component of the worldvolume gauge field is F_{x^{p-1},x^p}, and denote $\mathcal{F}_{x^{p-1},x^p}$ and F_{x^{p-1},x^p} by $\mathcal{F}$ and F. Our probe brane is extended along the $x^{p-1}x^p$ directions such that there are N' units of the worldvolume flux

$$\int dx^{p-1}dx^p F = \frac{2\pi}{T_f}N', \tag{24}$$

with $T_f = 1/2\pi\alpha'$ being the fundamental string tension.

The DBI action is calculated as

$$\begin{aligned} S_{DBI} = &-T_{8-p}\Omega_{6-p}R^{7-p}\int dtdx^{p-1}dx^p\rho^{6-p}\sqrt{h_pf_p^{-1}+\mathcal{F}^2h_p^{-1}} \\ &\times\sqrt{r^{-2}f_p^{-1}-r^{-2}\dot{r}^2-\frac{\dot{\rho}^2}{1-\rho^2}-(1-\rho^2)\dot{\phi}^2}, \end{aligned} \tag{25}$$

where the dot denotes the derivative with respect to t. Then the total action can be written as

$$S = \int dtdx^{p-1}dx^p\mathcal{L}, \tag{26}$$

where the lagrangian density $\mathcal{L}$ is given by

$$\begin{aligned} \mathcal{L} = &T_{8-p}\Omega_{6-p}R^{7-p} \\ &\times\Big\{-\rho^{6-p}\lambda_1\sqrt{r^{-2}f_p^{-1}-r^{-2}\dot{r}^2-\frac{\dot{\rho}^2}{1-\rho^2}-(1-\rho^2)\dot{\phi}^2}+\lambda_2(-1)^{p+1}\rho^{7-p}\dot{\phi}\Big\}. \end{aligned} \tag{27}$$

In Eq. (27) the functions λ_1 and λ_2 are defined as

$$\lambda_1 = \sqrt{h_pf_p^{-1}+\mathcal{F}^2h_p^{-1}}, \quad \lambda_2 = F\cos\varphi. \tag{28}$$

The dynamics of the system is determined by the standard hamiltonian analysis. For simplicity, we absorb the $(-1)^{p+1}$ sign in WZ term into the redefinition of $\dot{\phi}$ if necessary. The conjugate momenta are

$$\begin{aligned} \mathcal{P}_r &= \frac{\partial\mathcal{L}}{\partial\dot{r}} \equiv T_{8-p}\Omega_{6-p}R^{7-p}\lambda_1\pi_r, \\ \mathcal{P}_\rho &= \frac{\partial\mathcal{L}}{\partial\dot{\rho}} \equiv T_{8-p}\Omega_{6-p}R^{7-p}\lambda_1\pi_\rho, \\ \mathcal{P}_\phi &= \frac{\partial\mathcal{L}}{\partial\dot{\phi}} \equiv T_{8-p}\Omega_{6-p}R^{7-p}\lambda_1\pi_\phi, \end{aligned} \tag{29}$$

where we have defined

$$\begin{aligned} \pi_r &= \frac{\rho^{6-p}}{r^2}\frac{\dot{r}}{\sqrt{r^{-2}f_p^{-1}-r^{-2}\dot{r}^2-\frac{\dot{\rho}^2}{1-\rho^2}-(1-\rho^2)\dot{\phi}^2}}, \\ \pi_\rho &= \frac{\rho^{6-p}}{1-\rho^2}\frac{\dot{\rho}}{\sqrt{r^{-2}f_p^{-1}-r^{-2}\dot{r}^2-\frac{\dot{\rho}^2}{1-\rho^2}-(1-\rho^2)\dot{\phi}^2}}, \\ \pi_\phi &= (1-\rho^2)\rho^{6-p}\frac{\dot{\phi}}{\sqrt{r^{-2}f_p^{-1}-r^{-2}\dot{r}^2-\frac{\dot{\rho}^2}{1-\rho^2}-(1-\rho^2)\dot{\phi}^2}}+\Lambda\rho^{7-p}. \end{aligned} \tag{30}$$

In the third expression of Eq. (30), Λ is defined as

$$\Lambda = \frac{\lambda_1}{\lambda_2}. \tag{31}$$

The hamiltonian density is

$$\mathcal{H} = \dot{r}\mathcal{P}_r + \dot{\rho}\mathcal{P}_\rho + \dot{\phi}\mathcal{P}_\phi - \mathcal{L} \equiv T_{8-p}\Omega_{6-p}R^{7-p}\lambda_1 h, \tag{32}$$

where we defined the reduced hamiltonian h in analogy with the reduced ones for momenta. From Eqs. (27) and (30), h is calculated as

$$h = r^{-1} f_p^{-\frac{1}{2}} \left[r^2\pi_r^2 + \rho^{2(6-p)} + (1-\rho^2)\pi_\rho^2 + \frac{(\pi_\phi - \Lambda\rho^{7-p})^2}{1-\rho^2} \right]^{\frac{1}{2}}. \tag{33}$$

C. Giant graviton configurations

Here we will consider the solution of the equations of motion derived from the reduced hamiltonian Eq. (33). From Eq. (3) the coordinate ρ plays the role of the size of the system on S^{6-p} shpere. For this reason we look for the solution of the equations of motion with constant ρ which corresponds to the giant graviton configuration. The same problem was considered in Ref. [9] for the case of probe branes moving in the near-horizon Dp brane background. Comparing the right-hand side of Eq. (33) with the corresponding expression in Ref. [9], the same kind of arrangement is possible if the condition

$$\Lambda = 1, \tag{34}$$

is satisfied. Indeed, if this condition is satisfied, h can be expressed as

$$h = r^{-1} f_p^{-\frac{1}{2}} \left[\pi_\phi^2 + r^2\pi_r^2 + (1-\rho^2)\pi_\rho^2 + \frac{(\pi_\phi\rho - \rho^{6-p})^2}{1-\rho^2} \right]^{\frac{1}{2}}. \tag{35}$$

The condition Eq. (34) is equivalent to $\lambda_1 = \lambda_2$. Inserting Eqs. (4) and (23) into Eq. (28), we have

$$\lambda_1^2 = \cos^2\varphi F^2 + f_p^{-1}\left(F\sin\varphi - \frac{1}{\cos\varphi} \right)^2. \tag{36}$$

Comparing the Eq. (36) with the definition of λ_2 of Eq. (28), one can conclude that the condition $\Lambda = 1$ is equivalent to the following constant value of the worldvolume gauge field F

$$F = \frac{1}{\sin\varphi\cos\varphi} = 2\csc(2\varphi). \tag{37}$$

Substituting Eq. (37) into Eq. (36), one gets that λ_1 is also constant and is given by

$$\lambda_1 = \frac{1}{\sin\varphi}. \tag{38}$$

This value of the worldvolume gauge field can be considered as an ansatz which allows us a class of fixed size solutions of the equation of motion in which we are interested. From now on we assume that F is given by Eq. (37).

Now we are ready to find the configuration of the system with constant ρ. From Eq. (30), we have

$$\pi_\rho = 0. \tag{39}$$

Then, from the hamiltonian equation of motion for π_ρ , i.e. $\dot{\pi}_\rho = -\partial h/\partial\rho$, the last term on the right-hand side of Eq. (35) must vanish. For $p < 6$ this happens either for

$$\rho = 0, \tag{40}$$

or when π_ϕ is given by

$$\pi_\phi = \rho^{5-p}. \tag{41}$$

For $p = 6$, only Eq. (41) gives the constant ρ configuration. Since h does not depend on ϕ explicitly, π_ϕ is a constant of motion. Thus, for $p \neq 5$, Eq. (41) makes sense only when ρ is constant. Actually the constant value of ρ is determined by the value of π_ϕ. When $p = 5$, $\pi_\phi = 1$ regardless of the value of ρ.

Taking $\Lambda = 1$, from the last expression of Eq. (30), $\dot{\phi}$ is calculated as

$$\dot{\phi} = \frac{\pi_\phi - \rho^{7-p}}{1-\rho^2} \frac{\left[r^{-2}(f_p^{-1} - \dot{r}^2) - \frac{\dot{\rho}^2}{1-\rho^2}\right]^{\frac{1}{2}}}{\left[\pi_\phi^2 + \frac{(\pi_\phi\rho - \rho^{6-p})^2}{1-\rho^2}\right]^{\frac{1}{2}}}. \tag{42}$$

Since $\dot{\rho} = 0$, one can easily check that $\dot{\phi}$ and $\dot{r}$ satisfy

$$f_p(r^2\dot{\phi}^2 + \dot{r}^2) = 1, \tag{43}$$

whenever one of the two conditions in Eq. (40) or Eq. (41) is met. For the configurations we are considering, the last two terms inside the square root of the reduced hamiltonian Eq. (35) vanish and this configuration certainly minimizes the energy. Eq. (43) is the condition satisfied by a particle moving in the (r, ϕ) plane at $\rho = 0$ along a null trajectory in the metric Eq. (1). Thus the configurations have the characteristic of a massless particle i.e. the giant graviton. Note that $\rho = 0$ can be considered as the center of mass of the brane.

The momentum density $\mathcal{P}_\phi$ can be obtained from Eqs. (29) and (41) together with Eqs. (5), (16), (22) and (38)

$$\mathcal{P}_\phi = \frac{T_f}{2\pi} F N \rho^{5-p}, \tag{44}$$

where F is given in Eq. (37). The momentum p_ϕ and p_r can be obtained by integrating the momentum densities $\mathcal{P}_\phi$ and $\mathcal{P}_r$ over the $x^{p-1}x^p$ plane

$$p_\phi = \int dx^{p-1}dx^p\mathcal{P}_\phi, \qquad p_r = \int dx^{p-1}dx^p\mathcal{P}_r. \tag{45}$$

Using Eqs. (44) and (24), we have

$$p_\phi = NN'\rho^{5-p}. \tag{46}$$

For $p < 5$, the size of the wrapped brane increases with the momentum p_ϕ. Since $0 \le \rho \le 1$, the maximum value of the momentum is

$$p_\phi^{max} = NN' \tag{47}$$

for $\rho = 1$. It is known that the existence of the maximum angular momentum is the manifestation of the stringy exclusion principle [3]. For $p = 5$ the momentum p_ϕ is independent of the value of ρ. For $p = 6$, the value in Eq. (47) is actually the minimum. We will not consider $p = 6$ case significantly since we cannot define angular momentum on S^{6-p} for $p = 6$.

The energy of the giant graviton can be obtained by integrating the hamiltonian density $\mathcal{H}$ over the $x^{p-1}x^p$ plane

$$H_{GG} = f_p^{-\frac{1}{2}}\left(p_r^2 + \frac{p_\phi^2}{r^2}\right)^{\frac{1}{2}} = R^{\frac{p-7}{2}}\left(r^{7-p}p_r^2 + r^{5-p}p_\phi^2\right)^{\frac{1}{2}}. \tag{48}$$

We can study the equation of motion of this system by requiring the conservation of energy $H_{GG} = E$. From the first expression of Eq. (30) and Eq. (33), we have

$$\pi_r = f_p h\dot{r}. \tag{49}$$

Multiplying $T_{8-p}\Omega_{6-p}R^{7-p}\lambda_1$ on both sides and integrating over $x^{p-1}x^p$, we have

$$p_r = f_p H\dot{r} = \frac{R^{7-p}}{r^{7-p}}E\dot{r}. \tag{50}$$

By substituting Eq. (50) into the condition $H_{GG} = E$, we get

$$\dot{r}^2 + \frac{r^{7-p}}{R^{7-p}}\left(\frac{p_\phi^2}{E^2R^{7-p}}r^{5-p} - 1\right) = 0. \tag{51}$$

This equation determines the range of values that r can take. The second term in Eq. (51) must be negative or zero. The points at which this term is zero are the turning points of the system. For $p < 5$, these points are $r = 0$ and r_* with r_* defined as

$$(r_*)^{5-p} = \frac{E^2}{p_\phi^2}R^{7-p}, \tag{52}$$

and the range of r is $0 \le r \le r_*$. For $p = 5$, only $r = 0$ is the turning point and r is unrestricted. For $p = 6$, $r = 0$ turning point is missing and $r > r_*$.

We can also express r as a function of t by solving Eq. (51). Let's consider $p \neq 5$ case first. In this case we have

$$t - t_* = R^{\frac{7-p}{2}}\int \frac{dr}{r^{\frac{7-p}{2}}\sqrt{1-(\frac{r}{r_*})^{5-p}}}, \qquad (p \neq 5), \tag{53}$$

where t_* is a integration constant. The integral on the right-hand side of Eq. (53) can be done with the following change of variable

$$\left(\frac{r}{r_*}\right)^{5-p} = \cos^2\theta, \qquad (p \neq 5). \tag{54}$$

After the integral, we have

$$\left(\frac{r_*}{r}\right)^{5-p} = 1 + (5-p)^2 r_*^{5-p} R^{p-7} \left(\frac{t-t_*}{2}\right)^2, \qquad (p \neq 5). \tag{55}$$

Note that t_* is the value at which $r = r_*$. So we can take $t_* = 0$ without loss of generality. For $p < 5$, $r \to 0$ as $t \to \pm\infty$, i.e. the giant graviton falls asymptotically to the horizon. However, for $p = 6$, $r \to \infty$ as $t \to \pm\infty$, i.e. it always escapes to infinity. The $p = 5$ is a special case and easier to integrate and the result is

$$r = r_0 e^{\pm\frac{t}{R}\sqrt{1-\frac{p_\phi^2}{E^2R^2}}}, \qquad (p = 5). \tag{56}$$

The solution connects asymptotically the point $r = 0$ and $r = \infty$.

Similarly we can express ϕ as a function of t. We substitute the $r(t)$ expression into Eq. (43) to get $\dot{\phi}$ then we integrate it over t. The result for $p \neq 5$ is

$$\tan\left[\frac{5-p}{2}(\phi - \phi_*)\right] = \frac{5-p}{2}\left(\frac{r_*}{R}\right)^{\frac{5-p}{2}} \frac{t}{R}, \qquad (p \neq 5), \tag{57}$$

and for $p = 5$,

$$\phi = \phi_0 + \frac{p_\phi}{ER^2} t, \qquad (p = 5). \tag{58}$$

III. VIBRATION MODES OF GIANT GRAVITONS

In the previous section we have reviewed how the giant graviton picture appears in the near horizon background of D$(p-2)$-Dp branes. Here we will consider the perturbations of the giant gravitons from the equilibrium configurations. A small vibration of the brane can be described by defining spacetime coordinates $(r, \rho, \phi, x_k (k = 1, \cdots, p-2))$ as functions of the worldvolume coordinates $(t, x^{p-1}, x^p, \theta^1, \cdots, \theta^{6-p})$

$$\begin{aligned}
r &= r_0(t) + \epsilon\ \delta r(t, x^{p-1}, x^p, \theta^1, \cdots, \theta^{6-p}), \\
\rho &= \rho_0 + \epsilon\ \delta\rho(t, x^{p-1}, x^p, \theta^1, \cdots, \theta^{6-p}), \\
\phi &= \phi_0(t) + \epsilon\ \delta\phi(t, x^{p-1}, x^p, \theta^1, \cdots, \theta^{6-p}), \\
x_k &= \epsilon\ \delta x_k(t, x^{p-1}, x^p, \theta^1, \cdots, \theta^{6-p}), \qquad (k = 1, \cdots, p-2).
\end{aligned} \tag{59}$$

Here ρ_0 is a constant with $\rho_0 = 0$ or $\rho_0^{5-p} = \pi_\phi$. $r_0(t)$ and $\phi_0(t)$ are the solutions of the unperturbed equilibrium configuration found in the previous section. The action of the probe brane can be expanded in orders of ϵ as

$$S = \int dt dx^{p-1} dx^p d\theta^1 \cdots d\theta^{6-p} \Big(\mathcal{L}_0 + \mathcal{L}_1(\epsilon) + \mathcal{L}_2(\epsilon^2) + \cdots\Big). \tag{60}$$

Obviously $\mathcal{L}_0$ gives the zeroth order lagrangian density that we have used in Sec. II.

A. First order perturbation

First we expand the action to linear order in ϵ. We substitute Eq. (59) into the brane action Eq. (13) and a straightforward calculation gives

$$\begin{aligned}\mathcal{L}_1(\epsilon) &= \epsilon T_{8-p} R^{7-p}\sqrt{\hat{g}^{6-p}}\lambda_1 \\ &\times \Bigg[\delta\rho\Bigg\{-\frac{\rho_0^{7-p}\dot{\phi}_0^2}{\sqrt{\frac{r_0^{5-p}}{R^{7-p}}-\frac{\dot{r}_0^2}{r_0^2}-(1-\rho_0^2)\dot{\phi}_0^2}} \\ &\qquad -(6-p)\rho_0^{5-p}\sqrt{\frac{r_0^{5-p}}{R^{7-p}}-\frac{\dot{r}_0^2}{r_0^2}-(1-\rho_0^2)\dot{\phi}_0^2}+(7-p)\rho_0^{6-p}\dot{\phi}_0\Bigg\} \\ &+\delta\dot{\phi}\Bigg\{\frac{(1-\rho_0^2)\rho_0^{6-p}\dot{\phi}_0}{\sqrt{\frac{r_0^{5-p}}{R^{7-p}}-\frac{\dot{r}_0^2}{r_0^2}-(1-\rho_0^2)\dot{\phi}_0^2}}+\rho_0^{7-p}\Bigg\} \\ &+\frac{\delta r}{r_0}\Bigg\{-\frac{\rho_0^{6-p}}{\sqrt{\frac{r_0^{5-p}}{R^{7-p}}-\frac{\dot{r}_0^2}{r_0^2}-(1-\rho_0^2)\dot{\phi}_0^2}}\left(\frac{5-p}{2}\frac{r_0^{5-p}}{R^{7-p}}+\frac{\dot{r}_0^2}{r_0^2}\right)\Bigg\} \\ &+\frac{\delta\dot{r}}{\dot{r}_0}\Bigg\{\frac{\rho_0^{6-p}}{\sqrt{\frac{r_0^{5-p}}{R^{7-p}}-\frac{\dot{r}_0^2}{r_0^2}-(1-\rho_0^2)\dot{\phi}_0^2}}\frac{\dot{r}_0^2}{r_0^2}\Bigg\}\Bigg].\end{aligned} \tag{61}$$

If we substitute

$$\dot{\phi}_0^2 = r_0^{-2}(f_p^{-1}-\dot{r}_0^2), \tag{62}$$

obtained from Eq. (43), the square root in Eq. (61) is just $\rho_0\dot{\phi}_0$. Then one can easily check that the coefficient of $\delta\rho$ vanishes. The coefficient of $\delta\dot{\phi}$ is constant (ρ_0^{5-p}) and thus this term does not contribute to the variation of the action with fixed boundary values. The last term ($\delta\dot{r}$ term) can be written as

$$\delta\dot{r}\left(\frac{\rho_0^{5-p}\dot{r}_0}{\dot{\phi}_0 r_0^2}\right). \tag{63}$$

Integrating by parts with respect to t and neglecting the boundary terms, this term can be replaced by

$$-\delta r\frac{d}{dt}\left(\frac{\rho_0^{5-p}\dot{r}_0}{\dot{\phi}_0 r_0^2}\right). \tag{64}$$

Combining this term with the third term(δr term), the coefficient of δr is

$$\begin{aligned}&-\rho_0^{5-p}\left[\frac{1}{r_0\dot{\phi}}\left(\frac{5-p}{2}\frac{r_0^{5-p}}{R^{7-p}}+\frac{\dot{r}_0^2}{r_0^2}\right)+\frac{d}{dt}\left(\frac{\dot{r}_0}{\dot{\phi}_0 r_0^2}\right)\right] \\ =&-\rho_0^{5-p}\frac{1}{r_0\dot{\phi}_0}\left(\frac{5-p}{2}\frac{r_0^{5-p}}{R^{7-p}}-\frac{\dot{r}_0^2}{r_0^2}-\frac{\dot{r}_0}{r_0}\frac{\ddot{\phi}_0}{\dot{\phi}_0}+\frac{\ddot{r}_0}{r_0}\right).\end{aligned} \tag{65}$$

To simplify this expression we substitute the following equation,

$$\ddot{\phi}_0 = \frac{1}{\dot{\phi}_0}\Big(\frac{5-p}{2}\frac{r_0^{4-p}}{R^{7-p}}\dot{r}_0 + \frac{\dot{r}_0^3}{r_0^3} - \frac{\dot{r}_0\ddot{r}_0}{r_0^2}\Big), \tag{66}$$

obtained from Eq. (62), into Eq. (65), the coefficient of δr is calculated as

$$\begin{aligned} & -\rho_0^{5-p}\frac{1}{r_0\dot{\phi}_0^3}\Big[\Big(\frac{5-p}{2}\frac{r_0^{5-p}}{R^{7-p}} - \frac{\dot{r}_0^2}{r_0^2} + \frac{\ddot{r}_0}{r_0}\Big)\dot{\phi}_0^2 - \Big(\frac{5-p}{2}\frac{r_0^{3-p}}{R^{7-p}}\dot{r}_0^2 + \frac{\dot{r}_0^4}{r_0^4} - \frac{\dot{r}_0^2\ddot{r}_0}{r_0^3}\Big)\Big] \\ = & -\rho_0^{5-p}\frac{1}{r_0\dot{\phi}_0^3}\Big[\frac{5-p}{2}\frac{r_0^{2(5-p)}}{R^{2(7-p)}} - (6-p)\frac{r_0^{3-p}}{R^{7-p}}\dot{r}_0^2 - \frac{r_0^{4-p}}{R^{7-p}}\ddot{r}_0\Big]. \end{aligned} \tag{67}$$

We have used Eq. (62) in the the above equation to get the second line. This expression can be simplified further. Differentiating Eq. (51), we have

$$\ddot{r}_0 = -(6-p)\frac{p_\phi^2}{E^2 R^{2(7-p)}}r_0^{11-2p} + \frac{7-p}{2}\frac{r_0^{6-p}}{R^{7-p}}. \tag{68}$$

Substituting Eqs. (51) and (68) into Eq. (67), one can show that the square bracket is just zero. Thus, we find that the first order term in ϵ vanishes. This confirms that the zeroth order solution described in Sec. II is the right solution which minimizes the action.

B. Second order perturbation

Now we consider the second order term in ϵ. The second order term is calculated as

$$\begin{aligned} \mathcal{L}_2(\epsilon^2) = & -\frac{\epsilon^2}{2}T_{8-p}R^{7-p}\rho_0^{7-p}\lambda_1\omega_0\sqrt{\hat{g}^{6-p}} \\ \times\Big[& -\frac{1}{\rho_0^2(1-\rho^2)\omega_0^2}(\dot{\delta\rho})^2 + \sum_{i=1}^{6-p}\frac{1}{\rho_0^2(1-\rho^2)}\Big(\frac{\partial\delta\rho}{\partial\theta_i}\Big)^2\hat{g}^{\theta_i\theta_i} + \frac{1}{\lambda_1^2}\sum_{j=p-1}^{p}\frac{r_0^2}{1-\rho^2}\Big(\frac{\partial\delta\rho}{\partial x^i}\Big)^2 \\ & -\frac{1-\rho^2}{\rho_0^4\omega_0^2}(\dot{\delta\phi})^2 + \sum_{i=1}^{6-p}\frac{1-\rho_0^2}{\rho_0^2}\Big(\frac{\partial\delta\phi}{\partial\theta_i}\Big)^2\hat{g}^{\theta_i\theta_i} + \frac{1}{\lambda_1^2}\sum_{j=p-1}^{p}r_0^2(1-\rho_0^2)\Big(\frac{\partial\delta\phi}{\partial x^i}\Big)^2 \\ & -\frac{1}{\rho_0^2 r_0^2\omega_0^2}(\dot{\delta r})^2 + \sum_{i=1}^{6-p}\frac{1}{r_0^2\rho_0^2}\Big(\frac{\partial\delta r}{\partial\theta_i}\Big)^2\hat{g}^{\theta_i\theta_i} + \frac{1}{\lambda_1^2}\sum_{j=p-1}^{p}\Big(\frac{\partial\delta r}{\partial x^j}\Big)^2 \\ & \qquad + \frac{1}{\rho_0^2 r_0^2}\frac{5-p}{2}\Big(4-p-\frac{5-p}{2}\frac{1}{\rho_0^2}\Big)(\delta r)^2 \\ & + \sum_{k=1}^{p-2}\Big\{-\frac{1}{\rho_0^2}(\dot{\delta x_k})^2 + \sum_{i=1}^{6-p}\frac{\omega_0^2}{\rho_0^2}\Big(\frac{\partial\delta x_k}{\partial\theta_i}\Big)^2\hat{g}^{\theta_i\theta_i} + \frac{1}{\lambda_1^2}\sum_{j=p-1}^{p}\omega_0^2 r_0^2\Big(\frac{\partial\delta x_k}{\partial x^i}\Big)^2\Big\} \\ & + (5-p)\Big(-\frac{2}{\rho_0^3\omega_0}\delta\rho\dot{\delta\phi} + \frac{5-p}{\rho_0^3 r_0}\delta\rho\delta r + \frac{1-\rho_0^2}{\rho_0^4\omega_0 r_0}\dot{\delta\phi}\delta r\Big)\Big], \end{aligned} \tag{69}$$

where $\omega_0 = \dot{\phi}$.

Generally ω_0 and $\dot{r}_0$ in the above expression are functions of time, so we have to consider the time dependence of these variables in deriving the equations of motion. However, since our formalism is valid only for the near-horizon region ($r << R$), we can consider the perturbation around this region. Thus we consider ω_0 and r_0 as time independent values around the equilibrium configuration at $t = t^* = 0$. The equations of motion for this case are given by

$$\frac{1}{\rho_0^2(1-\rho_0^2)\omega_0^2}\ddot{\delta\rho} - \frac{1}{\rho_0^2(1-\rho_0^2)}\sum_{i=1}^{6-p}\frac{\partial}{\partial\theta_i}\Big(\frac{\partial\delta\rho}{\partial\theta_i}\hat{g}^{\theta_i\theta_i}\Big) - \frac{1}{\lambda_1^2}\frac{r_0^2}{1-\rho_0^2}\sum_{j=p-1}^{p}\frac{\partial}{\partial x^j}\Big(\frac{\partial\delta\rho}{\partial x^j}\Big)$$
$$-(5-p)\frac{1}{\rho_0^3\omega_0}\dot{\delta\phi} + \frac{(5-p)^2}{2}\frac{1}{\rho_0^3 r_0}\delta r = 0, \tag{70}$$

$$\frac{1-\rho_0^2}{\rho_0^4\omega_0^2}\ddot{\delta\phi} - \frac{1-\rho_0^2}{\rho_0^2}\sum_{i=1}^{6-p}\frac{\partial}{\partial\theta_i}\Big(\frac{\partial\delta\phi}{\partial\theta_i}\hat{g}^{\theta_i\theta_i}\Big) - \frac{1}{\lambda_1^2}r_0^2(1-\rho_0^2)\sum_{j=p-1}^{p}\frac{\partial}{\partial x^j}\Big(\frac{\partial\delta\phi}{\partial x^j}\Big)$$
$$+(5-p)\frac{1}{\rho_0^3\omega_0}\dot{\delta\rho} - \frac{5-p}{2}\frac{1-\rho_0^2}{\rho_0^4\omega_0 r_0}\dot{\delta r} = 0, \tag{71}$$

$$\frac{1}{\rho_0^2 r_0^2\omega_0^2}\ddot{\delta r} - \frac{1}{\rho_0^2 r_0^2}\sum_{i=1}^{6-p}\frac{\partial}{\partial\theta_i}\Big(\frac{\partial\delta r}{\partial\theta_i}\hat{g}^{\theta_i\theta_i}\Big) - \frac{1}{\lambda_1^2}\sum_{j=p-1}^{p}\frac{\partial}{\partial x^j}\Big(\frac{\partial\delta r}{\partial x^j}\Big)$$
$$+\frac{5-p}{2}\frac{1}{\rho_0^2 r_0^2}\Big(4-p-\frac{5-p}{2}\frac{1}{\rho_0^2}\Big)\delta r + \frac{(5-p)^2}{2}\frac{1}{\rho_0^3 r_0}\delta\rho + \frac{5-p}{2}\frac{1-\rho_0^2}{\rho_0^4 r_0\omega_0}\dot{\delta\phi} = 0, \tag{72}$$

$$\frac{1}{\rho_0^2}\ddot{\delta x}_k - \frac{\omega_0^2}{\rho_0^2}\sum_{i=1}^{6-p}\frac{\partial}{\partial\theta_i}\Big(\frac{\partial\delta x_k}{\partial\theta_i}\hat{g}^{\theta_i\theta_i}\Big) - \frac{1}{\lambda_1^2}\omega_0^2 r_0^2\sum_{j=p-1}^{p}\frac{\partial}{\partial x^j}\Big(\frac{\partial\delta x_k}{\partial x^j}\Big) = 0, \quad (k=1,\cdots,p-2). \tag{73}$$

We observe that δx_k perturbations decouple from $\delta\rho$, $\delta\phi$ and δr perturbations.

Let us Introduce the spherical harmonics Y_l on S^{6-p},

$$g^{\theta_i\theta_j}\frac{\partial}{\partial\theta_i}\frac{\partial}{\partial\theta_j}Y_l(\theta_1,\ldots,\theta_{6-p}) = -Q_l Y_l(\theta_1,\ldots,\theta_{6-p}), \tag{74}$$

where Q_l is the eigenvalue of the Laplace operator on the unit $6-p$ sphere given by

$$Q_l = l(l+5-p), \qquad l = 0,1,2,\cdots. \tag{75}$$

Choosing the harmonic oscillation, perturbations can be expressed as

$$\begin{aligned}
\delta\rho(t,x_{p-1},x_p,\theta_1,\ldots,\theta_{6-p}) &= \tilde{\delta}\rho e^{-i\omega t}e^{ik_{p-1}x_{p-1}}e^{ik_p x_p}Y_l(\theta_1,\ldots,\theta_{6-p}),\\
\delta\phi(t,x_{p-1},x_p,\theta_1,\ldots,\theta_{6-p}) &= \tilde{\delta}\phi e^{-i\omega t}e^{ik_{p-1}x_{p-1}}e^{ik_p x_p}Y_l(\theta_1,\ldots,\theta_{6-p}),\\
\delta r(t,x_{p-1},x_p,\theta_1,\ldots,\theta_{6-p}) &= \tilde{\delta}r e^{-i\omega t}e^{ik_{p-1}x_{p-1}}e^{ik_p x_p}Y_l(\theta_1,\ldots,\theta_{6-p}),\\
\delta x_k(t,x_{p-1},x_p,\theta_1,\ldots,\theta_{6-p}) &= \tilde{\delta}x_k e^{-i\omega t}e^{ik_{p-1}x_{p-1}}e^{ik_p x_p}Y_l(\theta_1,\ldots,\theta_{6-p}).
\end{aligned} \tag{76}$$

From Eq. (73), we find the frequency for δx_k perturbations as

$$\omega_{x_k}^2 = \omega_0^2\Big\{Q_l + \frac{\rho_0^2 r_0^2}{\lambda_1^2}(k_{p-1}^2+k_p^2)\Big\} \equiv \omega_0^2 Q'_l, \quad (k=1,\cdots,p-2). \tag{77}$$

The $\delta\rho$, $\delta\phi$ and δr perturbations are coupled and their normal modes are determined by the following matrix equation

$$\left(\begin{array}{ccc} \frac{1}{1-\rho_0^2}(Q'_l - \frac{\omega^2}{\omega_0^2}) & i\frac{5-p}{\rho_0}\frac{\omega}{\omega_0} & \frac{(5-p)^2}{2}\frac{1}{\rho_0 r_0} \\ -i\frac{5-p}{\rho_0}\frac{\omega}{\omega_0} & \frac{1-\rho_0^2}{\rho_0^2}(\rho_0^2 Q'_l - \frac{\omega^2}{\omega_0^2}) & i\frac{5-p}{2}\frac{1-\rho_0^2}{\rho_0^2 r_0}\frac{\omega}{\omega_0} \\ \frac{(5-p)^2}{2}\frac{1}{\rho_0^3 r_0} & -i\frac{5-p}{2}\frac{1-\rho_0^2}{\rho_0^4 r_0}\frac{\omega}{\omega_0} & \frac{1}{\rho_0^2 r_0^2}\Big(Q'_l + \frac{5-p}{2}(4-p-\frac{5-p}{2}\frac{1}{\rho_0^2}) - \frac{\omega^2}{\omega_0^2}\Big) \end{array} \right) \left(\begin{array}{c} \tilde{\delta\rho} \\ \tilde{\delta\phi} \\ \tilde{\delta r} \end{array} \right) = 0. \tag{78}$$

Defining $X \equiv \omega/\omega_0$, the normal modes of the coupled equations can be found from the following equation obtained from the determinant of the matrix

$$\begin{aligned} &(X^2 - Q'_l)(X^2 - \rho_0^2 Q'_l)\Big\{X^2 - Q'_l - \frac{n}{2}(n-1-\frac{n}{2\rho_0^2})\Big\} + \frac{n^4}{4}\frac{1-\rho_0^2}{\rho_0^2}(X^2 + \rho_0^2 Q'_l) \\ &\quad - n^2 X^2\Big\{X^2 - Q'_l - \frac{n}{2}(n-1-\frac{n}{2\rho_0^2})\Big\} - \frac{n^2}{4}\frac{1-\rho_0^2}{\rho_0^2}X^2(X^2 - Q'_l) = 0, \end{aligned} \tag{79}$$

where we defined $n \equiv 5-p$ for simplicity of the expression.

The condition for the giant graviton to be stable over the perturbation is that Eq. (79) have all real roots. The existence of imaginary part in ω means that the $e^{-i\omega t}$ term can grow exponentially, which means the instability of the configuration. We will check whether it has all real roots or not from the functional analysis. For this purpose, we define a function $f(y)$ such that

$$\begin{aligned} f(y) &= (y - Q'_l)(y - \rho_0^2 Q'_l)\Big\{y - Q'_l - \frac{n}{2}(n-1-\frac{n}{2\rho_0^2})\Big\} + \frac{n^4}{4}\frac{1-\rho_0^2}{\rho_0^2}(y + \rho_0^2 Q'_l) \\ &\quad - n^2 y\Big\{y - Q'_l - \frac{n}{2}(n-1-\frac{n}{2\rho_0^2})\Big\} - \frac{n^2}{4}\frac{1-\rho_0^2}{\rho_0^2}y(y - Q'_l) \\ &\equiv y^3 + c_2 y^2 + c_1 y + c_0, \end{aligned} \tag{80}$$

where $y = X^2 = \omega^2/\omega_0^2$ and c_i's are calculated as

$$\begin{aligned} c_0 &= -\rho_0^2 Q'^3_l - \frac{1}{2}n(n-1-\frac{n}{2\rho_0^2})\rho_0^2 Q'^2_l + \frac{1}{4}n^4(1-\rho_0^2)Q'_l, \\ c_1 &= (1+2\rho_0^2)Q'^2_l + n(n - \frac{1}{2} + \frac{n-1}{2}\rho_0^2)Q'_l + \frac{1}{4}n^3(n-2), \\ c_2 &= -(2+\rho_0^2)Q'_l - \frac{1}{4}n(5n-2). \end{aligned} \tag{81}$$

The condition for $X = \omega/\omega_0$ to have all real roots is equivalent to for $f(y) = 0$ to have all non-negative real roots. From the graphical analysis, this is satisfied if the following four conditions are satisfied. First of all, $df/dy = f'(y) = 3y^2 + 2c_2 y + c_1 = 0$ should have real solution, i.e.

$$\text{(i)} \qquad c_2^2 - 3c_1 \geq 0. \tag{82}$$

Let the two real roots of $f'(y) = 3y^2 + 2c_2 y + c_1 = 0$ be α and β ($\alpha < \beta$), then

$$\text{(ii)} \qquad f(\alpha)f(\beta) \le 0, \tag{83}$$

and the smaller one (α) should be positive

$$\text{(iii)} \qquad \alpha = \frac{-c_2 - \sqrt{c_2^2 - 3c_1}}{3} > 0. \tag{84}$$

The vertical-axis intercept $f(y=0)$ should be negative

$$\text{(iv)} \qquad f(y=0) = c_0 < 0. \tag{85}$$

First we consider the condition (iv). One can easily check that for $n = 0 (p = 5)$, $c_0 = -\rho_0^2 {Q'}_l^3$, thus the condition (iv) is automatically satisfied for all regions of $0 < \rho_0 \le 1$ and ${Q'}_l = l(l+n) + (\rho_0^2 r_0^2/\lambda_1^2)(k_{p-1}^2 + k_p^2) > 0$. For $n \ne 0$, the condition (iv) is satisfied for

$$\rho_0^2 > \frac{n^2({Q'}_l + n^2)}{4{Q'}_l^2 + 2n(n-1){Q'}_l + n^4}, \qquad (n \ne 0), \tag{86}$$

from the first expression of Eq. (81).

Secondly, let us examine the conditions (i) and (iii). These two are satisfied for any c_1 and c_2 with

$$c_1 > 0, \qquad c_2 < 0. \tag{87}$$

From Eq. (81), the conditions $c_1 > 0$ and $c_2 < 0$ are satisfied for

$$\rho_0^2 > -\frac{{Q'}_l^2 + n(n-\frac{1}{2}){Q'}_l + \frac{3}{4}n^3(n-2)}{2{Q'}_l^2 + \frac{1}{2}n(n-1){Q'}_l}, \tag{88}$$

$$\rho_0^2 > -\frac{2{Q'}_l + \frac{1}{4}n(5n-2)}{{Q'}_l}. \tag{89}$$

For $n = 0$ Eqs. (88) and (89) are is always satisfied for all values of $0 < \rho_0 \le 1$ and ${Q'}_l > 0$. For $n \ne 0$, for the given range of ρ_0 in Eq. (86), Eqs. (88) and (89) are automatically satisfied for all $0 < \rho_0 \le 1$ and ${Q'}_l > 0$.

Finally we consider the condition (ii). We can write down the condition (ii) explicitly in terms of c_i's. Using $3\alpha^2 + 2c_2\alpha + c_1 = 0$, we can write

$$f(\alpha) = \frac{2}{3}(c_1 - \frac{1}{3}c_2^2)\alpha + c_0 - \frac{c_1 c_2}{9}, \tag{90}$$

and same expression for $f(\beta)$. Then we have

$$\begin{aligned} f(\alpha)f(\beta) &= \left\{\frac{2}{3}(c_1 - \frac{1}{3}c_2^2)\alpha + c_0 - \frac{c_1c_2}{9}\right\}\left\{\frac{2}{3}(c_1 - \frac{1}{3}c_2^2)\beta + c_0 - \frac{c_1c_2}{9}\right\} \\ &= \frac{4}{9}(c_1 - \frac{1}{3}c_2^2)^2\alpha\beta + \frac{2}{3}(c_1 - \frac{1}{3}c_2^2)(c_0 - \frac{c_1c_2}{9})(\alpha+\beta) + (c_0 - \frac{c_1c_2}{9})^2. \end{aligned} \tag{91}$$

Since α and β are two real roots of $3y^2 + 2c_2 y + c_1 = 0$, substituting

$$\alpha + \beta = -\frac{2}{3}c_2, \quad \alpha\beta = \frac{1}{3}c_1, \tag{92}$$

the condition (ii) can be written as

$$\frac{4}{27}c_1(c_1 - \frac{1}{3}c_2^2)^2 + \frac{4}{9}c_2(c_1 - \frac{1}{3}c_2^2)(c_0 - \frac{c_1 c_2}{9}) + (c_0 - \frac{c_1 c_2}{9})^2 \leq 0. \tag{93}$$

For the given ranges of $0 < \rho_0 \leq 1$ and ${Q'}_l > 0$, this condition is always satisfied for all possible values of $n = 0, 1, 2, 3$ ($p = 5, 4, 3, 2$).

We can summarize the above result as follows. For $n = 0 (p = 5)$, the vibration modes are all real for all range of $0 < \rho_0 \leq 1$. This means that the giant graviton configurations are stable for all values of $0 < \rho_0 \leq 1$. For $n \neq 0 (p \neq 5)$, the vibration modes are all real for

$$\frac{n^2({Q'}_l + n^2)}{4{Q'}_l^2 + 2n(n-1){Q'}_l + n^4} < \rho_0^2 \leq 1, \qquad (n \neq 0). \tag{94}$$

So the giant graviton configurations are stable for this range of ρ_0.

IV. DISCUSSION

We studied the stability of the giant gravitons in the string theory background with NSNS B field. We consider the perturbation from the stable configurations generated by D$(p-2)$-D(p) branes for $2 \leq p \leq 5$. The vibration modes for x_k's$(k = 1, \cdots, p-2)$ are decoupled and the frequencies are all real. The vibration modes for ρ , ϕ and r are coupled. For $p = 5$, they are stable independent of the size of the brane. For $p \neq 5$, we calculated the range of the size of the brane where they are stable. In the limit when the angular momentum on S^{6-p} is large, i.e. l is large, the condition in Eq. (94) becomes $0 < \rho_0^2 \leq 1$. This means that the giant graviton configurations are stable for large angular momentum regardless of the size of the branes. We would like to emphasize that Eq. (34) is the crucial condition in our calculation. It has been discussed in Ref. [9] that one can draw the giant graviton picture whenever this condition is met.

In the previous work of the vibration modes of giant gravitons in the dilatonic backgrounds [13], the perturbation along the transverse(r) direction was not considered. Only ρ and ϕ perturbations were considered and they are coupled. Their normal modes are determined by a 2×2 matrix equation. If we turn off the NSNS B field by setting $\varphi = 0 (h_p = 1)$, Eq. (1) reduces to the geometry of the dilatonic D(p) brane. So with $\delta r = 0$, we have from Eq. (78)

$$\begin{pmatrix} \frac{1}{1-\rho_0^2}(Q_l - \frac{\omega^2}{\omega_0^2}) & i\frac{5-p}{\rho_0}\frac{\omega}{\omega_0} \\ -i\frac{5-p}{\rho_0}\frac{\omega}{\omega_0} & \frac{1-\rho_0^2}{\rho_0^2}(\rho_0^2 Q_l - \frac{\omega^2}{\omega_0^2}) \end{pmatrix} \begin{pmatrix} \tilde{\delta}\rho \\ \tilde{\delta}\phi \end{pmatrix} = 0. \tag{95}$$

This gives the frequencies of two modes

$$\frac{\omega_\pm^2}{\omega_0^2} = \frac{1}{2}\Big[(1+\rho_0^2)Q_l + (5-p)^2 \pm \sqrt{(5-p)^4 + 2(5-p)^2(1+\rho_0^2)Q_l + Q_l^2(1-\rho_0^2)^2}\Big], \tag{96}$$

which is exactly the same result obtained in Ref. [13].

In our perturbation, we considered ω_0 and r_0 as time independent because the background configuration is valid only in near-horizon region. The brane motion in the transverse direction generally induces a cosmological evolution of the brane universe, called the mirage cosmology [14]. The relevance of giant gravitons to the mirage cosmology was pointed by Youm [15]. Further studies on this issue is expected in future.

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